**Final Exam - 12/05/2019**

**The exam is take home.**

1.The cylindrical bar shown in fig. consists of two segments; the left segment is clamped at point R, Φ1(0) = 0. The left segment of length L is a solid circular bar of radius RO, while the right segment of length L is a hollow circular bar of inner radius Ri. A moment Q1 is applied at point T.

(1) Determine the twist angle at point T.

(2) Determine the equivalent torsional stiffness, H, for the complete bar, defined as H = Φ1(2L)/Q1.

(3) Determine ratio of maximum shear stress in the two sections. (50 points)



2. Measured displacements for the three loading cases. Load are measured in kN,

displacements in mm.

|  |  |  |  |
| --- | --- | --- | --- |
|  | P1 = 1.5 | P2 = 1.0  | P3 = 0.5  |
| Δ1 | 10.9 | 18.3 | 14.6 |
| Δ2 | 27.7 | 59.1 | 51.1 |
| Δ3 | 43.1 | 104 | 98.5 |

lower beam under a mid-span concentrated load.

(1) Show that the lower beam can be replaced by a concentrated spring of stiffness constant k0 = 6H0/L3.

(2) Find the exact solution of the problem from the solutions of the governing differential equations and associated boundary conditions for both upper and lower beams. Replace the interaction between the beams by a force X, yet unknown. The magnitude of this force is found by equating the displacements of the upper and lower beams at point A.

(3) Plot the distribution of transverse displacement for both beams. Use L/a = 2.



 Fig: Two beam assembly under transverse load

(4) Plot the distribution of bending moment for both beams.

(5) Plot the distribution of shear force for both beams. (50 points)